JOURNAL OF APPLIED POLYMER SCIENCE VOL. 7, PP. 493-506 (1963)

Origins of Normal Stress in Capillary Jets of Newtonian and Viscoelastic Liquids

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In the steady laminar flow of a Newtonian liquid in a capillary tube the pressure gradient in the tube gives rise to a shear stress in the fluid in the direction of the flow and opposite to the pressure gradient. More complicated and specifically viscoelastic liquids may be shown to develop normal stresses in addition to the shear stress. Depending upon the equation of state employed for the liquid, one can predict radial or axial, tensile or compressive, stresses from theoretical studies. However, although normal stresses have been measured in Couette flow and in flow between a plate and a cone of small angle, for example, there is as yet no satisfactory direct method of measurement of normal stress *within the capillary*. If a measurement technique could be developed it would provide a useful adjunct to the methods now being employed in the study of normal stresses in viscoelastic liquids.

Associated with the presence of normal stresses in capillary flow is the expansion or contraction of a jet of a viscoelastic liquid upon ejection from a capillary nozzle into a low-viscosity medium such as air. The authors have shown[1,2] that the presence of a relaxing axial tensile stress in such a jet in some cases may, if the ejection velocity is not too high, cause the jet to expand to more than twice the diameter of the capillary nozzle. One source of the tensile stress is the normal stress developed within the capillary, which, when the constraint of the capillary wall is removed, is free to relax.

There have been attempts to utilize the expansion–contraction phenomenon as a measure of this normal stress. Gaskins and Philippoff[4] measured the decrease in velocity associated with expansion and attempted to evaluate the normal stress by means of an energy balance, but they erroneously omitted the not inconsiderable, but difficult to evaluate, dissipation of energy accompanying the change of flow velocity profile, from that occurring in the tube, to the flat profile downstream from the tube exit. Their interpretation of the results is meaningless, therefore. More recently, Metzner and co-workers[5] reported that they had obtained normal stress measurements using jets of carboxymethylcellulose (CMC) in water and

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polyisobutylene (PIB) in Decalin by measurement of expansion and the use of a momentum balance, which circumvented the difficulty of the dissipation term in the energy balance. These investigators did not recognize, however, that effects other than the normal stress developed in the capillary can contribute to the expansion–contraction phenomenon.

The present authors have shown[2,6] that, as either a viscoelastic or a Newtonian liquid is ejected and the constraint of the tube wall is removed, the liquid is no longer restricted to one-dimensional simple shear flow, and normal stresses are developed *outside* the capillary as a result of the change from the velocity profile in the capillary to the flat profile downstream from the tube exit. In viscoelastic jets an elastic reaction caused by the change in rate of shear during the velocity profile relaxation external to the capillary tube must be included as well. Other factors affecting the expansion–contraction phenomenon are the effects of surface tension and inertial forces. Metzner's analysis correctly accounted for the latter but ignored the former. Whether "remembered" entrance stresses contribute to the observed stress may be argued, but if they are present they are easily eliminated from consideration by the use of long enough capillary nozzles. Entrance stresses will not be discussed further in this paper.

In previous publications[1-3,6] the authors have described the expansion–contraction phenomenon in Newtonian and viscoelastic jets and have presented a rational analysis for it which leads to the measurement of a relaxing axial tensile stress whose value at the nozzle is only partially determined by an *internal* normal stress. The purpose of this paper is to examine the relative magnitudes of the individual contributions to the total tensile stress at the nozzle exit, with the idea of extracting that part contributed by the *internal* normal stress.

THE SURFACE TRACTION

The analysis of the expansion–contraction phenomenon allows contribution of surface traction to the observed tensile stress to be calculated. Its magnitude may be obtained from the correct form[3] of eq (9) of Ref. 2 for either Newtonian or viscoelastic jets:

$$T_\sigma = \rho \bar{U}^2(2\chi/\mathrm{We}^2) = 2\chi(\sigma/d_0) \tag{1}$$

where ρ is the fluid density, $\bar{U}$ is the average ejection velocity in the nozzle, χ is the ratio of final jet diameter to the nozzle diameter d_f/d_0, σ is the surface tension of the fluid, and We is the Weber number $(d_0\rho\bar{U}^2/\sigma)^{1/2}$. Figure 1 is a plot of T_σ versus σ/d_0 for several values of χ in the range occurring in the authors' experiments. For all practical values of σ/d_0 the contribution of the surface traction is less than 10^4 dynes. For $\chi = 1$ the surface traction is the same as obtained by Gavis and Gill[7] by considering the surface tension contribution to the wave propagation velocity on a jet.

For comparison purposes, corrected forms[3] of Figures 3 and 4 of Ref. 2 are reproduced here as Figures 2 and 3. The surface traction has

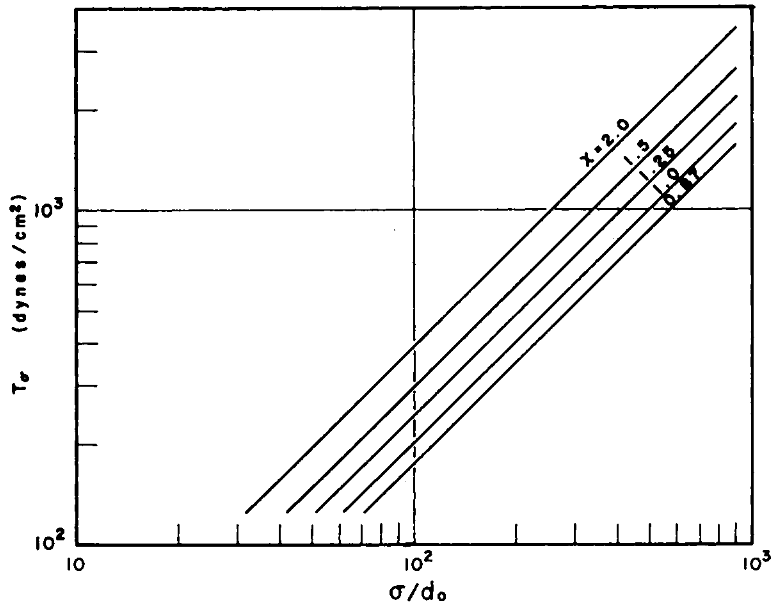

Fig. 1. Surface traction as a function of σ/d_0 for various values of the expansion ratio, χ.

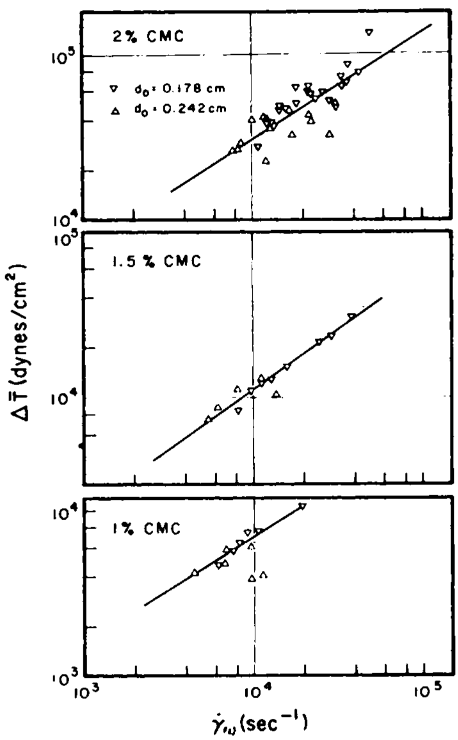

Fig. 2. Measured tensile stress, exclusive of the surface traction, as a function of shear rate at the wall, for jets of CMC in water.

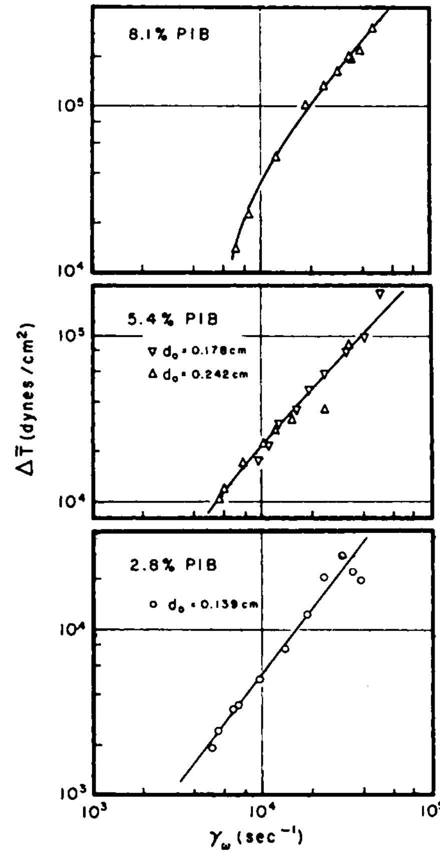

Fig. 3. Measured tensile stress, exclusive of the surface traction, as a function of shear stress at the wall, for jets of PIB in xylene.

been removed from the plotted stresses. It has been shown[2] that expansion-contraction experiments yield only the *difference* in average stress between the nozzle and a downstream point of measurement where the velocity profile has become flat; hence the labeling of the ordinate as $\Delta \bar{T}$. The stress difference is plotted against the rate of shear at the wall, $\dot{\gamma}_w$, which is directly proportional to $\bar{U}$ and inversely proportional to d_0.

It is immediately apparent that for the CMC jets (σ = 72 dynes/cm.) and for at least the 2.8% PIB jets (σ = 30 dynes/cm.), in nozzles of the order of 2 mm. I.D., the surface traction is an appreciable fraction of the total measured stress, especially at low ejection velocities.

RELAXATION OF THE TENSILE STRESS

That expansion of a capillary jet can occur is due mainly to the relaxation of the axial tensile stress; the contribution of the surface traction has been seen to be smaller than the stress. In Newtonian jets stress relaxation arises only because of the relaxation of the velocity profile; in viscoelastic jets the *internal* normal stress also must relax.

The rate of relaxation and the jet velocity determine the length of the jet along which expansion or contraction occurs: for a rapid relaxation rate, expansion or contraction will occur close to the nozzle; for slow enough relaxation, expansion or contraction may occur for a considerable length along the jet.[1] Since expansion to a maximum constant diameter occurred within 1 or 2 mm. of the nozzle in Newtonian jets at low Reynolds numbers, the velocity profile and the stress must have relaxed extremely rapidly. At higher Reynolds numbers where, because of the influence of inertial forces, the jets contracted, diameter change was observed to occur as far as a centimeter from the nozzle, indicating slower relaxation than at lower Reynolds numbers. Nevertheless, considering that velocities ranged up to 400 cm./sec. at the highest Reynolds numbers, the relaxation rate was still rapid.

In those viscoelastic jets which have been studied,[2] the relaxation rate, although not as rapid as in Newtonian jets, was large enough to cause all observed expansion (at lower velocities) and contraction (at the highest velocities) to occur at a centimeter, more or less, from the nozzle. Unfortunately, point values of the stress along the jet, and thus the rate of relaxation, cannot be obtained from expansion–contraction experiments, for one needs to know the mean square velocity across the jet at every point. As has already been pointed out,[2] this is as yet unobtainable.

Transverse wave propagation experiments,[7] however, showed that, at least in CMC jets at velocities greater than 500 cm./sec., after a large rapid change in tensile stress close to the nozzle, there remained a small, slowly relaxing, residual stress, over and above the surface traction, at very large distances from the nozzle. The wave experiments could not measure the stress at the nozzle. A combination of wave and expansion–contraction experiments, however, will give a complete stress history of the jet.

Goren and Gavis[8] showed that, for the appearance of nodal points from which stress information may be obtained, the total tensile stress at the nozzle, $T_{0,\text{ tot}}$, had to be small enough that $T_{0,\text{ tot}}/\rho U_f^2 \ll 1$, where U_f is the velocity of the jet far enough downstream from the nozzle for it to be constant.

Wave experiments were performed, in the manner described by Gill and Gavis,[7] with the fluids for which expansion–contraction data were obtained.[2] With those CMC jets of high enough concentration that they were stable under the influence of the nozzle vibration, it was necessary to use ejection velocities greater than 500 cm./sec. in order to obtain nodes. In the case of PIB, no distinct nodal points could be obtained on stable jets, even with velocities up to 1000 cm./sec. One is, then, unable to obtain a complete stress history of CMC jets below 500 cm./sec. or of PIB jets below 1000 cm./sec., or even greater. Because an expansion–contraction experiment measures the *difference* between the tension at the nozzle and at some point downstream where the velocity profile has become flat,[6] and one

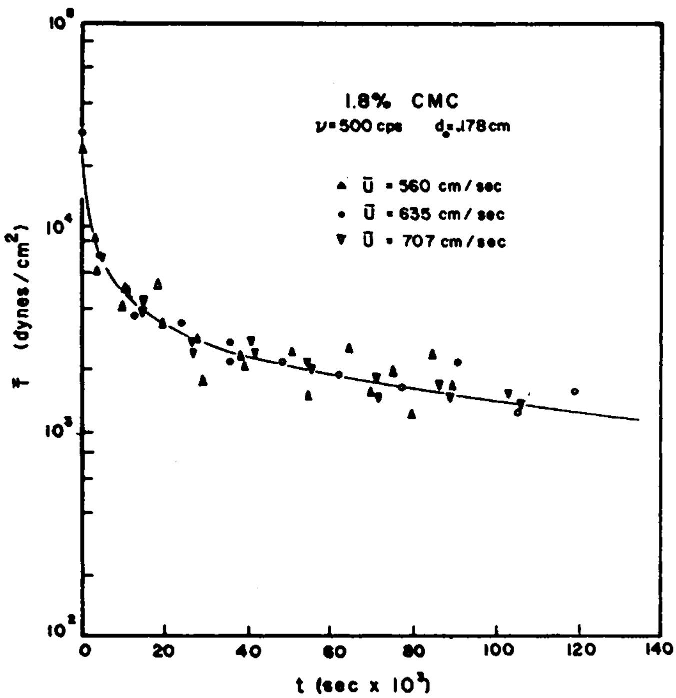

Fig. 4. Point values of tensile stress along the jet, exclusive of the surface traction, as a function of time since a jet element has kept the nozzle, for 1.8% CMC in water.

has no direct measure of the residual tensile stress at the downstream point of measurement of the jet diameter one, therefore, cannot know directly the absolute magnitude of the stress at the nozzle for such jets.

Figure 4 is a plot, taken from wave propagation experiments, of tensile stress in a 1.8% CMC jet, excluding surface traction, versus the time to reach any point on the jet. Stress was plotted against time rather than position so that the data of different velocities would be on the same basis. The two points on the vertical axis were obtained from expansion–contraction data. Although they represent the *difference* in stress between the nozzle and the downstream point of measurement, they have been plotted as absolute stresses here, for, as examination of the figure will show, the magnitude of the residual stress is only a few per cent of the difference in stress and, considering the experimental errors involved, will be undetectable in an expansion-contraction experiment.

For CMC jets at low velocity or for jets of such materials as PIB in xylene, with which wave experiments are not feasible, one must rely upon other means of estimating the residual stress. If measurements of the diameter of a jet far enough along its horizontal trajectory (but still not so far that the diameter is affected by gravity) show that for comparatively large distances there is no measurable change in diameter, one can assume that, unless the relaxation rate is improbably slow, the residual stress is small compared with the initial stress. This was found to be so in all the PIB and CMC jets to which this method could be applied, i.e., in jets of sufficient velocity that curvature in the trajectory and the effect of gravity was slight for a reasonable distance. One may, then, as a reasonable approximation, take the residual stress to be small and consider the stress difference measured by an expansion-contraction experiment to be the absolute stress at the nozzle exit for these jets.

Finally, in the wave propagation experiments, Gill and Gavis[7] found that the residual stress was apparently independent of ejection velocity, whereas the stress at the nozzle *did* depend upon velocity. It is obvious from Figure 4 that the reason for the apparent independence is that the experimental scatter is at least as large as the difference in stress for the comparatively small velocity range employed.

THE PROFILE RELAXATION STRESS

To ascertain the magnitude of the velocity profile relaxation stress at the nozzle exit for Newtonian jets, one may start with the final form of the momentum equation:[3,6]

$$1/\chi^2 + 2\chi/\mathrm{We}^2 = {}^4/_3 - 8N/\mathrm{Re} \tag{2}$$

$$N \equiv (2/\bar{U}d_0)\int_0^{d_0/2} (\partial u/\partial z)_0 r\, dr \qquad \text{dimensionless} \tag{3}$$

where $(\partial u/\partial z)_0$ is the axial velocity gradient at any radial position at the nozzle and Re is the Reynolds number $(d_0\bar{U}\rho/\mu)$, with μ the viscosity of the liquid. Equation (2) may be rearranged into:

$$1/\chi^2 + 2\chi/\mathrm{We}^2 = {}^4/_3 - (\bar{\tau}'_{zz})_0/\rho\bar{U}^2 \tag{4}$$

where $(\bar{\tau}'_{zz})_0$, the average normal profile relaxation stress at the nozzle, is given by:

$$(\bar{\tau}'_{zz})_0 = (8/d_0^2)\int_0^{d_0/2} \tau'_{zz} r\, dr = (16\mu/d_0^2)\int_0^{d_0/2} (\partial u/\partial z)_0 r\, dr \tag{5}$$

$$(\tau'_{zz})_0 = 2\mu(\partial u/\partial z)_0 \tag{6}$$

The stress $(\tau'_{zz})_0$ may be related to the shear stress at the wall τ_w for a Newtonian fluid in capillary flow:

$$\tau_w = \mu(\partial u/\partial r)_w = -\,8\mu\bar{U}/d_0 \tag{7}$$

by combination of eqs. (3), (5), and (7) to give:

$$(\bar{\tau}'_{zz})_0 = (8\mu\bar{U}/d_0)N = -\,\tau_w N \tag{8}$$

Although the integral N is theoretically derivable from the equations of Newtonian fluid mechanics, no method of solving those equations for the free boundary problem of the capillary jet has as yet been found, and it is necessary to proceed empirically.

In Figure 2(*a*) of Ref. 6 the expansion ratio χ is plotted as a function of Re; in Figure 2(*b*) the same data are plotted but the surface traction is

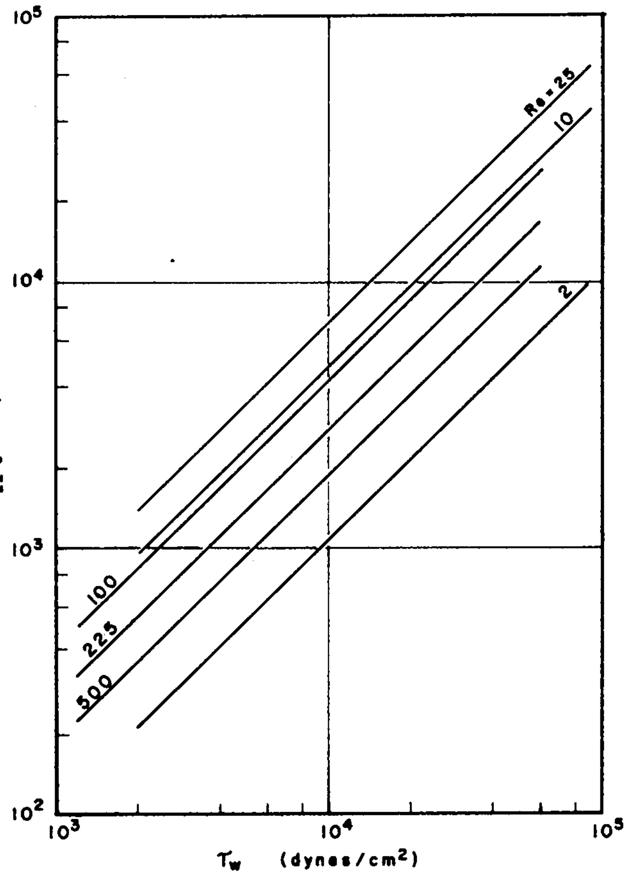

Fig. 5. Profile relaxation stress in Newtonian jets, exclusive of the surface traction, as a function of shear stress at the wall, for various values of the Reynolds number.

removed. Although the latter plot is in the form of $(1/\chi^4 + 4/\mathrm{We}^2\chi)^{-1/4}$ against Re, made in accordance with the energy equation for the jet, it is easy to show by a binomial expansion that for Weber number magnitudes at least as large as those used in the authors' experiments, the following obtains:

$$(1/\chi^4 + 4/\mathrm{We}^2\chi)^{-1/4} \cong (1/\chi^2 + 2\chi/\mathrm{We}^2)^{-1/2}$$

so that the plot is the same as that which would have resulted if the data were plotted as $(1/\chi^2 + 2\chi/\mathrm{We}^2)^{-1/2}$ against Re according to the momentum equation. The data of Figure 2(*b*) of Ref. 6 may then be represented by:

$$(1/\chi^2 + 2\chi/\mathrm{We}^2)^{-1/2} = 1.06 - 0.19 \exp\{-60/\mathrm{Re}^{1/2}\} \tag{9}$$

which is the alternative form of eq. (16) of Ref. 6. Comparison with eq. (2) gives, for N:

$$N = \mathrm{Re}/8\{4/3 - [1.06 - 0.19 \exp\{-60/\mathrm{Re}^{1/2}\}]^{-2}\} \tag{10}$$

Equation (10), together with eq. (8), enables one to determine $(\bar{\tau}'_{zz})_0$ as a function of τ_w and Re. Because τ_w and Re are independently variable, $(\bar{\tau}'_{zz})_0$ can only be presented as a function of either τ_w or Re with the other as a parameter. A plot of $(\bar{\tau}'_{zz})_0$ as a function of $|\tau_w|$ at various values of Re is given in Figure 5. Although $(\bar{\tau}'_{zz})_0$ increases linearly with $|\tau_w|$, its dependence upon Re is more complicated; for any $|\tau_w|$ the stress will be a maximum at a value of Re near 25.

For non-Newtonian fluids the situation, of course, is much more complicated since, besides the even greater difficulty involved in determining the non-Newtonian equivalent of N analytically, there also is no satisfactory experimental method of determining it: the non-Newtonian shear behavior of the fluids was confounded in the jet experiments by their viscoelastic behavior. One may, however, obtain an estimate of the relative importance of the profile relaxation stress for viscoelastic liquids in the following manner. For the range of Re which occurred in the Newtonian jet experiments (between about 2 and 200) N varied between 0.1 and 0.7. The profile relaxation stress, therefore, was always smaller than the shear stress at the wall. One may assume that a similar situation occurs in viscoelastic jets and, in the absence of the relation between them, consider that the shear stress at the wall is an upper limit to the profile relaxation stress.

Table I takes the data of Figures 2 and 3 and compares the values of $\mathcal{T}_0$, the stress after removal of the surface traction, with $|\tau_w|$ computed from the average ejection velocity by means of:

$$-\tau_w = \kappa\left(\frac{3n+1}{4n}\frac{8\bar{U}}{d_0}\right)^n \tag{12}$$

where κ and n are the consistency index and flow index (listed for these fluids in Table II, Ref. 2) of the flow "power law" which the liquids were found to obey in the range of shear rates employed.

TABLE I
Comparison of $\mathcal{T}_0$ with $|\tau_w|$

Fluid	$\vert\tau_w\vert$	$\mathcal{T}_0$
CMC		
1.0%	1,450	4,320
	2,730	10,200
1.5%	3,030	8,500
	6,000	31,700
2.0%	5,150	13,900
	7,950	34,400
PIB		
2.8%	1,400	2,030
	4,910	20,600
5.4%	4,200	11,400
	11,700	186,000
8.1%	10,100	15,400
	20,400	299,000

For each liquid the extreme values of $|\tau_w|$ are listed. One may conclude that at low values of $|\tau_w|$, corresponding in general to low ejection velocities, profile relaxation may contribute an appreciable fraction of the tensile stress, $\bar{T}_0$. At high values of $|\tau_w|$ (high ejection velocities) the profile relaxation contribution is considerably smaller; in fact, it may be much smaller than appears in Table I because it is to be expected that at the higher ejection velocities, where the Reynolds numbers are higher, the profile relaxation stress will be considerably smaller than $|\tau_w|$, as in corresponding Newtonian jets. At high ejection velocities one *may* be justified in ignoring the profile relaxation stress. At these velocities the jets will expand only slightly or they will contract.

THE EXTERNAL NORMAL STRESS

This contribution to the observed tensile stress in viscoelastic jets arises because of the elastic reaction to the velocity profile relaxation. Oldroyd[9] showed how an elastic reaction can be developed in a viscoelastic liquid whenever there occurs a time rate of change of shear rate, as must exist in an element of the jet moving from the nozzle to a point downstream where the velocity profile is flat. A simple model will be chosen to illustrate how this comes about. In this model the stress is a linear function of the strain rate and the time rate of change of strain rate. For the shear components this is written:

$$\tau'^{ij} = \mu_1(\dot{\gamma}^{ij} + \theta_1 \partial\dot{\gamma}^{ij}/\partial t) \tag{13}$$

where μ_1 and θ_1 are constants with the dimensions of viscosity and time, respectively, and $\dot{\gamma}^{ij}$ is the strain rate (here, the shear rate). This model is similar to that used by Oldroyd except that a term containing the time rate of change of the stress has been omitted. The final result will have fewer terms, be simpler, and be no less *illustrative* than if the extra term were present.

Oldroyd generalized eq. (13) for arbitrary types of flow and for arbitrarily large strain rates. This involved the replacement of the time derivative in eq. (13) by a "convected" time derivative, written $\mathfrak{D}/\mathfrak{D}t$, and the choice of either contravariant of covariant indices for the stress and strain rate tensors. Because Oldroyd showed that the former choice gave results more in accord with observation than the latter it has been adopted here. Then:

$$\tau'^{ij} = \mu_1(\dot{\gamma}^{ij} + \theta_1 \mathfrak{D}\dot{\gamma}^{ij}/\mathfrak{D}t)$$

$$\mathfrak{D}\dot{\gamma}^{ij}/\mathfrak{D}t = \partial\dot{\gamma}^{ij}/\partial t + v^k(\partial\dot{\gamma}^{ij}/\partial x^k) - (\partial v^i/\partial x^k)\dot{\gamma}^{kj} - (\partial v^j/\partial x^k)\dot{\gamma}^{ik} \tag{14*}$$

where the summation convention is used, v^i are the components of the velocity vector and x^i are the Cartesian position coordinates.

* In this and in the following equations the stress components have their signs reversed from those in the rest of this paper. The sign convention used here is the more usual one; the opposite convention was used in the rest of the paper in order to conform to the designation of the tensile stress as positive in earlier publications.

For fully developed, steady flow *within* the capillary:

$$\dot{\gamma}^{ij} = {}^1/_2 \begin{pmatrix} 0 & 0 & dv^z/dr \\ \mathrm{C} & 0 & 0 \\ dv^z/dr & 0 & 0 \end{pmatrix} \tag{15}$$

$$\mathfrak{D}\dot{\gamma}^{ij}/\mathfrak{D}t = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -(dv^z/dr)^2 \end{pmatrix} \tag{16}$$

When these are inserted into eq. (14), their results, for the stress components:

$$\tau'^{rz} = \mu_1(dv^z/dr) \tag{17a}$$

$$\tau'^{zz} = -2\mu_1\theta_1(dv^z/dr)^2 \tag{17b}$$

$$\tau'^{rr} = 0 \tag{17c}$$

The quantity $(dv^z/dr)^2$ is obtainable from the equations of motion and continuity for capillary flow; τ'^{zz} is the internal normal stress. It is to be noted that Oldroyd's generalization of a linear model introduces nonlinear dependence of the stress on the strain rate.

In the jet the velocity is not limited to the axial direction alone. To conserve space, the equivalents of eqs. (15) and (16) will not be written. The final result for τ'^{zz}, which is the component of primary interest, is:

$$\tau'^{zz} = 2\mu_1(\partial v^z/\partial z) - 2\mu_1\theta_1(\partial v^z/\partial r)^2 + 2\mu_1\theta_1[2v^z(\partial^2 v^z/\partial z^2) + 2v^r(\partial^2 v^z/\partial z\partial r) - (\partial v^z/\partial r)(\partial v^r/\partial z)] \tag{18}$$

The first and second terms on the right will be recognized as the point values of the profile relaxation and *internal* normal stresses. The additional terms on the right represent the elastic reaction to the profile relaxation, the *external* normal stress. Although the equations of motion and continuity provide additional relationships by which the velocities and velocity gradients may be determined, the nonlinearity of the system precludes doing this practically. Had the additional term in Oldroyd's model been included, τ'^{zz} would have been positive in eq. (17b) with a different constant factor; eq. (18) would have been a differential equation for τ'^{zz}, with the second term on the right positive and with a different constant factor.

There is, of course, no way of knowing whether this, or even Oldroyd's more complete, model is adequate to describe the viscoelastic liquids used in the experiments; the development here is only to show how the external normal stress *may* originate.

THE INTERNAL NORMAL STRESS

Although rigorous arguments cannot be advanced until adequate equations of state for the liquids used are found, one may proceed tentatively, assuming the correctness of the convected derivative formulation for the generalization of simple equations of state.

It has already been noted that at high ejection velocities the profile relaxation stress is appreciably smaller than the *internal* plus *external* normal stresses for the PIB and CMC jets investigated. Now, at high ejection velocities the jets expand but slightly, or contract somewhat less than 10% of the nozzle diameter. This occurs over an axial distance which is larger than the jet radius, so that v^r and its derivatives with respect to r and z are smaller than v^z and its corresponding derivatives. Also, $(\partial v^z/\partial z)$ is smaller than $(\partial v^z \partial r)$. Therefore, at high ejection velocities the largest term in eq. (18) is the term $2\mu_1\theta_1(\partial v^z/\partial r)^2$, provided θ is not too small. One may conclude that, if the liquid is sufficiently viscoelastic and the ejection velocity is high enough, the largest single contribution to the observed tensile stress should be the *internal* normal stress. At low ejection velocities, where large expansion occurs, this should *not* be so. Pending solutions to the system of equations for the velocities and the stresses one must remember that these conclusions are tentative, and that the internal normal stresses which may be measured in high ejection velocity experiments are, at best, approximate values for the actual stresses.

CONCLUSIONS

In this and in previous publications the authors have described an investigation of the expansion-contraction phenomenon in capillary jets of Newtonian and viscoelastic liquids. Aside from the intrinsic interest this curious and heretofore inadequately studied phenomenon engendered, the investigation was aimed at developing from the observations a method of obtaining normal stress data for viscoelastic liquids in a simple shear flow, in order to provide one more tool in the attack on the normal stress problem.

It must be concluded that useful normal stress data for the simple shear field of the capillary cannot easily be obtained from expansion contraction data in viscoelastic liquid jets. For, although a relaxing normal stress is able to be determined, the stress desired, the *internal* normal stress, is only one component of the measured stress. The analysis was able to account for the contribution of the surface traction and the effect of inertial forces. It could not, however, account adequately for the profile relaxation stress, nor could it account adequately for the *external* normal stress, except possibly at high ejection velocities. In addition, a residual stress at the downstream point of measurement may remain in the jet and must be accounted for.

The phenomenon remains a fascinating one, but not quite as simple of interpretation as has sometimes been naïvely assumed. The difficulty lies in the complicated nonlinear system of equations which must be solved even for the Newtonian jet. Until solutions for the equations are found, expansion and contraction of capillary jets of viscoelastic liquids, as a method of obtaining a measure of the normal stress developed during flow of a viscoelastic liquid in a capillary tube, should be used only with extreme caution and with full realization of the approximate nature of the results obtained. The expansion–contraction phenomenon, unfortunately, turns out to be not as useful a tool as one would like it to be.

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Synopsis

The individual contributions to the observed normal tensile stress in capillary jets of Newtonian and viscoelastic liquids, measured by means of expansion-contraction experiments described in earlier publications of the authors, is discussed. The surface tension contribution is calculated. The contribution from the relaxing velocity profile is determined empirically for the Newtonian jet; it is estimated for the viscoelastic jet. The *internal* normal stress, developed in the capillary, and the *external* normal stress, developed beyond the capillary nozzle exit, are described, but not in a quantitative manner. Because of the complexity of the equations describing the viscoelastic jet, these stresses are not yet able to be separated in the general case. At high ejection velocities, however, the profile relaxation and the external normal stress should be smaller than the internal normal stress if the liquid is highly viscoelastic. One should, however, be fully aware of the approximate nature of the results when expansion-contraction experiments are used to obtain a measure of the normal stress developed during flow in a capillary. Such experiments, unfortunately, are not as useful as one would like them to be.

Résumé

On discute les contributions individuelles à la force de tension normale observée dans les écoulements capillaires des liquides Newtoniens et viscoélastiques, mesurées à l'aide d'expériences d'expansion–contraction décrites dans les précédentes publications des auteurs. On a calculé la contribution de la tension superficielle. On a déterminé empiriquement la contribution due au profil de la vitesse de relaxation pour l'écoulement newtonien; on l'a estimée pour l'écoulement viscoélatique. On décrit la force *interne* normale, développée dans le capillaire, et la force *externe* normale, développée au delà de l'orifiçe de la sortie du capillaire, mais cette description n'est pas quantitative. Du fait de la complexité de l'équation relative à l'écoulement viscoélastique, ces forces ne peuvent être encore séparées dans le cas général. Aux fortes vitesses d'éjection, cependant, le profil de rélaxation et la force externe normale sont plus faibles que la force interne normale si le liquide est fortement viscoélastique. On pourrait, cependant, être complètement renseigné sur la nature approximative des résultats quand les essais d'expansion–contraction sont employés pour obtenir la mesure de la force normale développée durant l'écoulement dans un capillaire. De tels essais ne sont malheureusement pas aussi utiles qu'on le désirerait.

Zusammenfassung

Die individuellen Beiträge zu der beobachteten Normal-Zugspannung in Kapillarstrahlen von Newtonschen und viskoelastischen Flüssigkeiten wurden mittels der in früheren Veröffentlichungen des Autors beschriebenen Expansions–Kontraktions-Versuchen gemessen und werden diskutiert. Der Beitrag der Oberflächenspannung wird berechnet. Der Beitrag aus der Relaxation des Geschwindigkeitsprofils wird für den Newtonschen Strahl empirisch bestimmt; er wird für den viskoelastischen Strahl geschätzt. Die innere Normalspannung, die sich in der Kapillare ausbildet, und die äussere Normalspannung, die sich ausserhalb der Kapillardüse ausbildet, werden, wenn auch nicht quantitativ, beschrieben. Wegen der Kompliziertheit der Gleichungen für den viskoelastischen Strahl können diese Spannungen bis jetzt für den allgemeinen Fall noch nicht getrennt werden. Bei hoher Ausstossgeschwindigkeit sollte aber die Profilrelaxation und die äussere Normal spannung für stark viskoelastische Flüssigkeiten kleiner sein als die innere Normalspannung. Man sollte sich jedoch bei einer Anwendung von Expansions–Kontraktions-Versuchen zur Ermittlung eines Masses für die während des Fliessens in einer Kapillare entstehende Normalspannung des Näherungscharakters der Ergebnisse voll bewusst sein. Solche Versuche sind leider nicht so brauchbar, wie man es gerne wünschen würde.

Received September 26, 1961